

Another Odd Thing About Unparticle Physics

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Abstract

The peculiar propagator of scale invariant unparticles has phases that produce unusual patterns of interference with standard model processes. We illustrate some of these effects in $e^+e^- \rightarrow \mu^+\mu^-$.

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1 Introduction

In a previous paper [1], I argued that a scale invariant sector that decouples at a large scale is associated with “unparticles” whose production might be detectable in missing energy and momentum distributions. In this note, we consider some of the leading virtual effects of unparticles. In particular, we write down the unparticle propagator and consider the interference between standard model amplitudes and amplitudes involving virtual unparticles. As emphasized long ago by Eichten, Lane and Peskin [2], this kind of interference can be a sensitive probe of high-energy processes. In particular, the interference terms are effects of leading nontrivial order in the small couplings of unparticles to standard model particles, the same order as the production cross-sections considered in [1]. We will find that the peculiar phases associated with the unparticle propagator in the time-like region give rise to unusual patterns of interference which depend dramatically on the scaling dimension of the unparticles.

In this note, as in [1], we assume that the very high energy theory contains the fields of the standard model and the fields of a theory with a nontrivial IR fixed point, which I call \mathcal{BZ} (for Banks-Zaks [3]) fields. The two sets interact through the exchange of particles with a large mass scale $M_{\mathcal{U}}$. Below $M_{\mathcal{U}}$, there are nonrenormalizable couplings between standard model fields and Banks-Zaks fields suppressed by powers of $M_{\mathcal{U}}$. The renormalizable couplings of the \mathcal{BZ} fields then produce dimensional transmutation and the scale-invariant unparticle fields emerge below an energy scale $\Lambda_{\mathcal{U}}$. In the effective theory below the scale $\Lambda_{\mathcal{U}}$ the \mathcal{BZ} operators match onto unparticle operators, and the nonrenormalizable interactions match onto a new set of interactions between standard model and unparticle fields with small coefficients. We make crucial use of the simplifications that result by working to lowest nontrivial order in the small couplings of unparticles fields to standard model fields in the effective field theory below $\Lambda_{\mathcal{U}}$. This allows us reliably to calculate some important quantities without having to understand in detail what unparticles look like. We will return to some of these questions at the end of the paper.

To illustrate the interesting properties of the unparticle propagator, we consider the example of the low energy effect of the following interaction terms.

$$\frac{C_{V\mathcal{U}} \Lambda_{\mathcal{U}}^{k+1-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k} \bar{e} \gamma_{\mu} e O_{\mathcal{U}}^{\mu} + \frac{C_{A\mathcal{U}} \Lambda_{\mathcal{U}}^{k+1-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k} \bar{e} \gamma_{\mu} \gamma_5 e O_{\mathcal{U}}^{\mu} \quad (1)$$

where the unparticle operator is hermitian and transverse,

$$\partial_{\mu} O_{\mathcal{U}}^{\mu} = 0. \quad (2)$$

I stress that this is just an example. Unparticle operators with different tensor structures can be dealt with in a similar way.

In the notation of [1], the transverse 4-vector unparticle propagator is given by

$$\begin{aligned}
& \int e^{iPx} \langle 0 | T(O_{\mathcal{U}}^\mu(x) O_{\mathcal{U}}^\nu(0)) | 0 \rangle d^4x \\
&= i \frac{A_{d_{\mathcal{U}}}}{2\pi} \int_0^\infty (M^2)^{d_{\mathcal{U}}-2} \frac{-g^{\mu\nu} + P^\mu P^\nu / P^2}{P^2 - M^2 + i\epsilon} dM^2 \\
&= i \frac{A_{d_{\mathcal{U}}}}{2} \frac{-g^{\mu\nu} + P^\mu P^\nu / P^2}{\sin(d_{\mathcal{U}}\pi)} (-P^2 - i\epsilon)^{d_{\mathcal{U}}-2}
\end{aligned} \tag{3}$$

where

$$A_{d_{\mathcal{U}}} = \frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}} \frac{\Gamma(d_{\mathcal{U}} + 1/2)}{\Gamma(d_{\mathcal{U}} - 1) \Gamma(2d_{\mathcal{U}})} \tag{4}$$

We can check this odd-looking result by finding the discontinuity across the cut for $P^2 > 0$.

$$\begin{aligned}
& i \frac{A_{d_{\mathcal{U}}}}{2} \frac{-g^{\mu\nu} + P^\mu P^\nu / P^2}{\sin(d_{\mathcal{U}}\pi)} (P^2)^{d_{\mathcal{U}}-2} \left((-1 - i\epsilon)^{d_{\mathcal{U}}-2} - (-1 + i\epsilon)^{d_{\mathcal{U}}-2} \right) \\
&= i \frac{A_{d_{\mathcal{U}}}}{2} \frac{-g^{\mu\nu} + P^\mu P^\nu / P^2}{\sin(d_{\mathcal{U}}\pi)} (P^2)^{d_{\mathcal{U}}-2} (e^{-i(d_{\mathcal{U}}-2)\pi} - e^{i(d_{\mathcal{U}}-2)\pi}) \\
& i \frac{A_{d_{\mathcal{U}}}}{2} \frac{-g^{\mu\nu} + P^\mu P^\nu / P^2}{\sin(d_{\mathcal{U}}\pi)} (P^2)^{d_{\mathcal{U}}-2} (-2i \sin(d_{\mathcal{U}}\pi)) = A_{d_{\mathcal{U}}} \left(-g^{\mu\nu} + P^\mu P^\nu / P^2 \right) (P^2)^{d_{\mathcal{U}}-2}
\end{aligned} \tag{5}$$

in agreement with the arguments of [1].

As (5) shows, the non-trivial phases along the physical cut in (3) play a necessary role in reproducing the scale invariance. We will find that these phases, even more than the nontrivial scaling itself, produce unique physical effects in interference. These peculiar interference effects are the key results in this paper. We explore a few of these below for the explicit example of (1).

It is important to note that while the discontinuity across the cut is not singular for integer $d_{\mathcal{U}} > 1$, the propagator (3) is singular because of the $\sin(d_{\mathcal{U}}\pi)$ in the denominator. I believe that this is a real effect. These integer values describe multiparticle cuts and the mathematics is telling us that we should not be trying to describe them with a single unparticle field. For this reason we will focus on $1 < d_{\mathcal{U}} < 2$, and we will find that the virtual effect of unparticles are gentlest away from the integer boundaries.

Let us first compute the cross section for $e^+e^- \rightarrow \mu^+\mu^-$ in the the presence of the interactions (1). It is convenient to rescale the dimensional coefficients to the Z mass, and define the dimensionless coefficients

$$c_{V\mathcal{U}} = \frac{C_{V\mathcal{U}} \Lambda_{\mathcal{U}}^{k+1-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k M_Z^{1-d_{\mathcal{U}}}} \quad c_{A\mathcal{U}} = \frac{C_{A\mathcal{U}} \Lambda_{\mathcal{U}}^{k+1-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k M_Z^{1-d_{\mathcal{U}}}} \tag{6}$$

Then (ignoring the lepton masses) the square of the invariant matrix element can be written as (where $q^2 = s$ is the square of the total center of mass energy and θ is the angle

of the μ^- from the e^- direction in the center of mass)

$$|\mathcal{M}|^2 = 2(q^2)^2 \left[\left(|\Delta_{VV}(q^2)|^2 + |\Delta_{AA}(q^2)|^2 + |\Delta_{VA}(q^2)|^2 + |\Delta_{AV}(q^2)|^2 \right) (1 + \cos^2 \theta) \right. \\ \left. + \left(\text{Re}(\Delta_{VV}^*(q^2) \Delta_{AA}(q^2)) + \text{Re}(\Delta_{VA}^*(q^2) \Delta_{AV}(q^2)) \right) 4 \cos \theta \right] \quad (7)$$

where

$$\Delta_{xy}(q^2) \equiv \sum_{\substack{j=\gamma \\ Z, \mathcal{U}}} d_{xj}^e d_{yj}^{\mu *} \Delta_j(q^2) \quad \text{where } x, y = V \text{ or } A \quad (8)$$

with the ds given in the following table

d_{xj}	γ	Z	\mathcal{U}
V	e	$\frac{e}{\sin \theta \cos \theta} (-1/4 + \sin^2 \theta)$	$\frac{c_{V\mathcal{U}}}{M_Z^{d_{\mathcal{U}}-1}}$
A	0	$\frac{e/4}{\sin \theta \cos \theta}$	$\frac{c_{A\mathcal{U}}}{M_Z^{d_{\mathcal{U}}-1}}$

(9)

and the Δ_j s being the γ , Z and \mathcal{U} propagators,

Δ_j	γ	Z	\mathcal{U}
=	$\frac{1}{q^2}$	$\frac{1}{(q^2 - M_Z^2 + iM_Z\Gamma_Z)}$	$\frac{A_{d_{\mathcal{U}}}}{2 \sin(d_{\mathcal{U}}\pi)} (q^2)^{d_{\mathcal{U}}-2} e^{-i(d_{\mathcal{U}}-2)\pi}$

(10)

We have tacitly assumed in (9) that the unparticle interactions are lepton-flavor-blind, so that we do not have to keep track of the e and μ superscripts on the cs , and we will continue to assume this in the graphs below. But (7) is entirely general and does not depend on this assumption.

As a first example of the interesting structure of (7), consider the total cross section in the LEP region. We are used to thinking that the Z pole is not a good place to look for interference with the effects of small non-renormalizable interactions because the amplitude is dominantly imaginary on the pole. This prejudice is not warranted for unparticle interactions. The unparticle amplitude can interfere with both the real and imaginary parts of the standard model and can therefore contribute both on and off the pole.

It is instructive to begin by assuming $c_{V\mathcal{U}} = 0$ (remember, we are taking the same c for e and μ) and considering the total cross section. Because the vector coupling vanishes, the interference between the unparticle exchange amplitude and the photon decay amplitude does not contribute to the total cross section, so we expect only interference with Z exchange. In figure 1, I show the fractional change in the total cross section for small non-zero $c_{A\mathcal{U}}$ for various values of $d_{\mathcal{U}}$ between 1 and 2. The dominant effect as expected is the interference

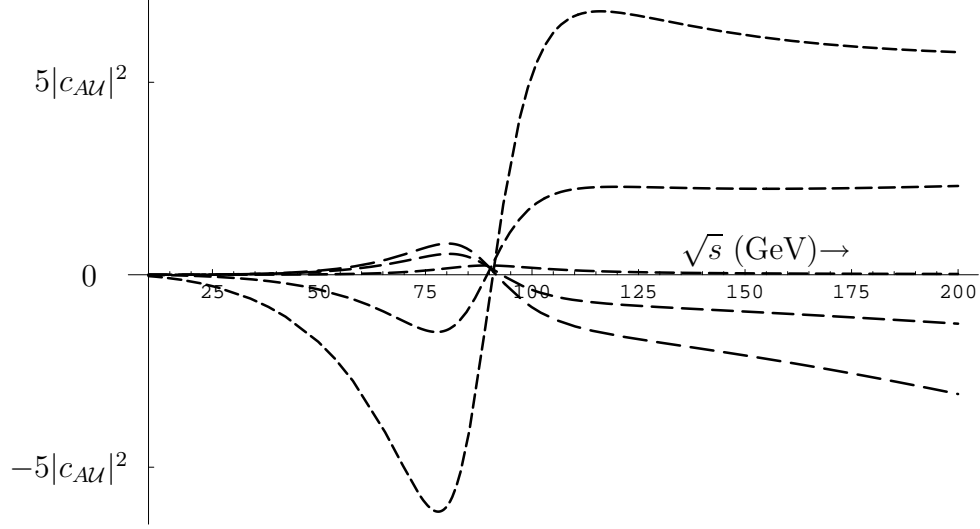


Figure 1: The fractional change in total cross-section for $e^+e^- \rightarrow \mu^+\mu^-$ versus \sqrt{s} for $d_U = 1.1, 1.3, 1.5, 1.7$ and 1.9 for non-zero c_{AU} and $c_{VU} = 0$. The dash-length increases with d_U .

term proportional to a single power of $|c_{AU}|^2$. But the striking thing about this graph is how sensitively the result depends on the value of d_U . We can understand qualitatively what is going on by thinking about the phase of the unparticle propagator along the physical cut which is

$$\phi_{d_U} = -(d_U - 1)\pi \quad (11)$$

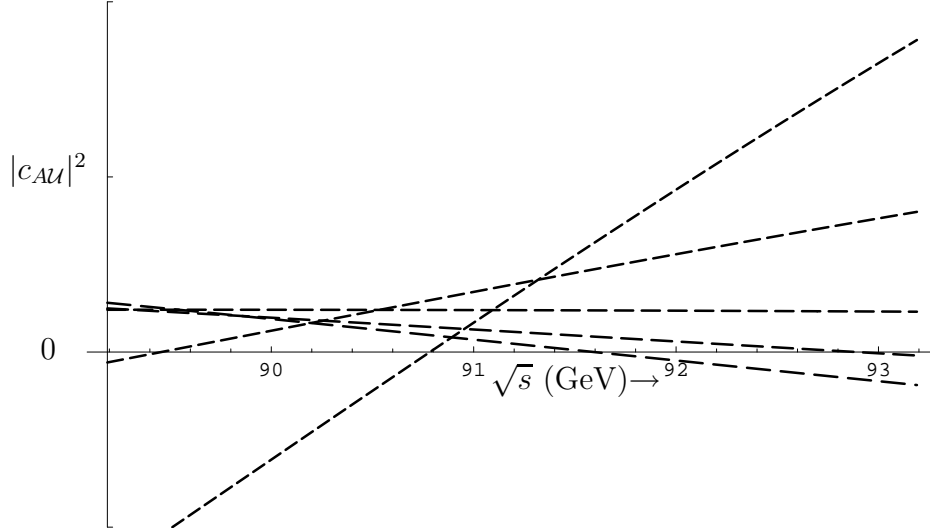


Figure 2: The fractional change in total cross-section for $e^+e^- \rightarrow \mu^+\mu^-$ versus \sqrt{s} for $d_U = 1.1, 1.3, 1.5, 1.7$ and 1.9 for non-zero c_{AU} and $c_{VU} = 0$. The dash-length increases with d_U . Note the different scales compared to figure 1.

The real part of (11) is positive for $1 < d_U < 3/2$ and negative for $3/2 < d_U < 2$. The

real part of $1/(q^2 - M_Z^2 + iM_Z\Gamma_Z)$ is negative below the Z pole and positive above. Thus away from the Z pole, where the imaginary part of $1/(q^2 - M_Z^2 + iM_Z\Gamma_Z)$ is small, we expect destructive (constructive) interference below (above) the pole for $1 < d_U < 3/2$, and vice-versa for $3/2 < d_U < 1$. Near the Z pole, the situation is complicated, as illustrated in figure 2 because both real and imaginary parts contribute to the interference.

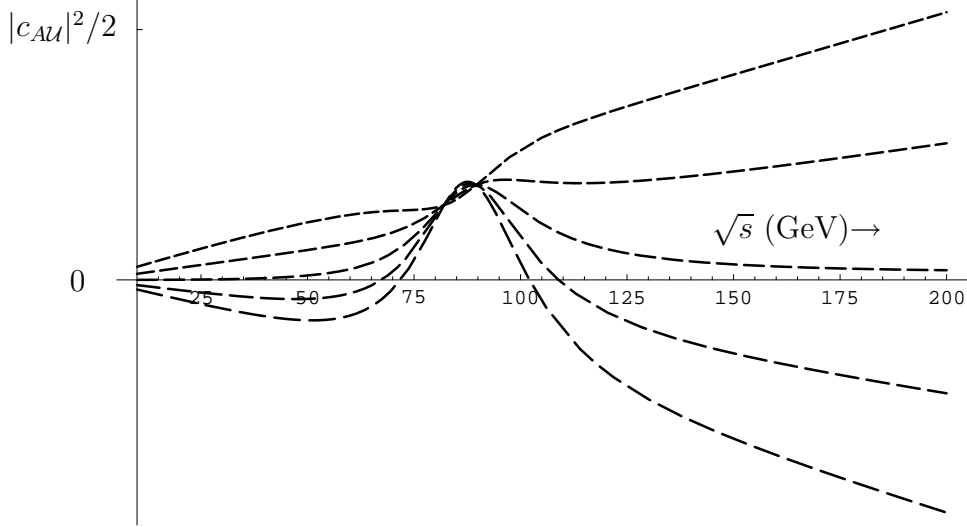


Figure 3: The fractional change in total cross-section for $e^+e^- \rightarrow \mu^+\mu^-$ versus \sqrt{s} for $d_U = 1.48, 1.49, 1.5, 1.51$ and 1.52 for non-zero c_{AU} and $c_{VU} = 0$. The dash-length increases with d_U . Note the different vertical scale compared to figure 1.

The situation simplifies in a very interesting way for $d_U = 3/2$. In this case, the phase from (11) is $\phi_{d_U} = -\pi/2$, so the unparticle amplitude interferes only with the imaginary part of the Z -exchange amplitude. This is a smaller effect than we see for values of d_U very different from $3/2$ because it is proportional to the Z width, rather than $q^2 - M_Z^2$. It gives constructive interference that peaks on the Z pole and goes to zero far from the pole. This is shown on a different scale in figure 3. Here I have also included a few values of d_U close to $3/2$, for comparison.

Having seen how things work for purely axial vector unparticle couplings, let us now consider what the total cross section looks like for a vector coupling. Now we expect interference with the photon-exchange amplitude, and because the vector part of the leptonic coupling of the Z is (by an “accident” of the value of $\sin^2\theta$) very small, there is very little interference with the Z -exchange amplitude. Now we expect constructive interference for $1 < d_U < 3/2$ and destructive interference for $3/2 < d_U < 1$. The result is shown in figure 4. The dip at the Z pole arises simply because we are plotting a fractional change and the large contribution from the pole is in the denominator.

The unparticle interference in the matrix element (7) also gives rise to a complicated pattern of changes in the front-back asymmetry

$$\frac{\sigma_f - \sigma_b}{\sigma_f + \sigma_b} = \frac{3}{8} \left(\frac{\text{Re}(\Delta_{VV}^*(q^2) \Delta_{AA}(q^2)) + \text{Re}(\Delta_{VA}^*(q^2) \Delta_{AV}(q^2))}{|\Delta_{VV}(q^2)|^2 + |\Delta_{AA}(q^2)|^2 + |\Delta_{VA}(q^2)|^2 + |\Delta_{AV}(q^2)|^2} \right) \quad (12)$$

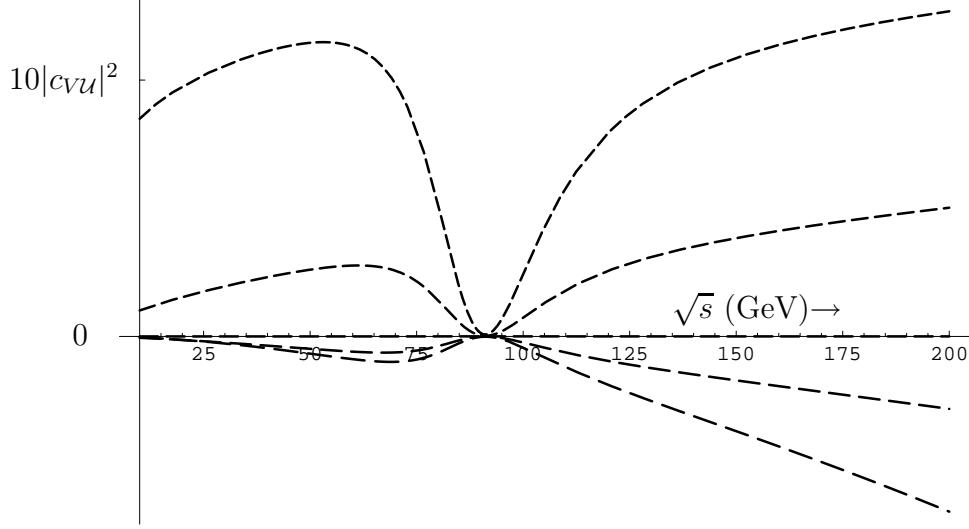


Figure 4: The fractional change in total cross-section for $e^+e^- \rightarrow \mu^+\mu^-$ versus \sqrt{s} for $d_U = 1.1, 1.3, 1.5, 1.7$ and 1.9 for non-zero c_{VU} and $c_{AU} = 0$. The dash-length increases with d_U .

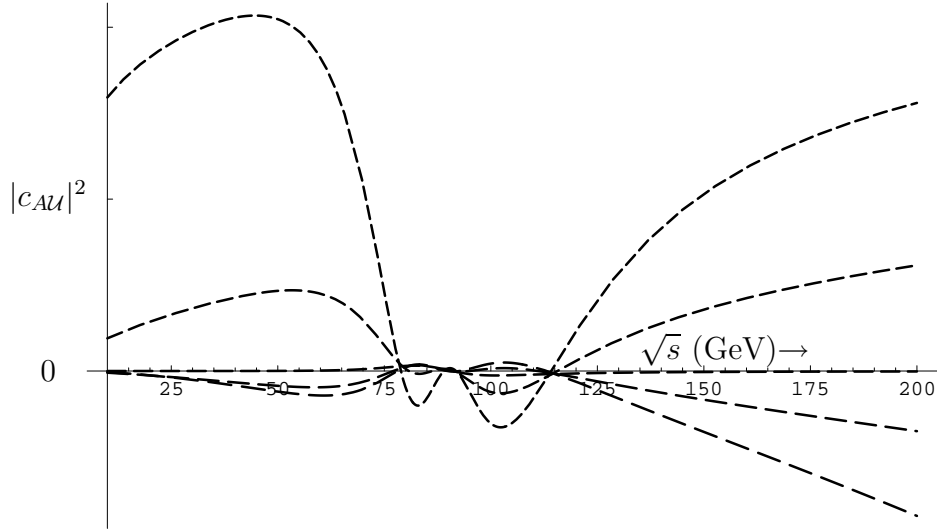


Figure 5: The change in the front-back asymmetry for $e^+e^- \rightarrow \mu^+\mu^-$ versus \sqrt{s} for $d_U = 1.1, 1.3, 1.5, 1.7$ and 1.9 for non-zero c_{AU} and $c_{VU} = 0$. The dash-length increases with d_U .

This is shown in figures 5 and 6. As for the total cross section, the effect for $d_U = 3/2$ is smaller and concentrated at the Z pole. In figures 5 and 6, we focus down on values of $d_U \approx 3/2$.

I hope I have convinced the reader that the unparticle propagator in the time-like region has interesting properties that force us to reexamine many of our preconceived notions about interference. Working to lowest non-trivial order in the couplings of the unparticles in the effective low energy theory, we can make detailed predictions of the form of interference between time-like unparticle exchange amplitudes and standard model amplitudes even though

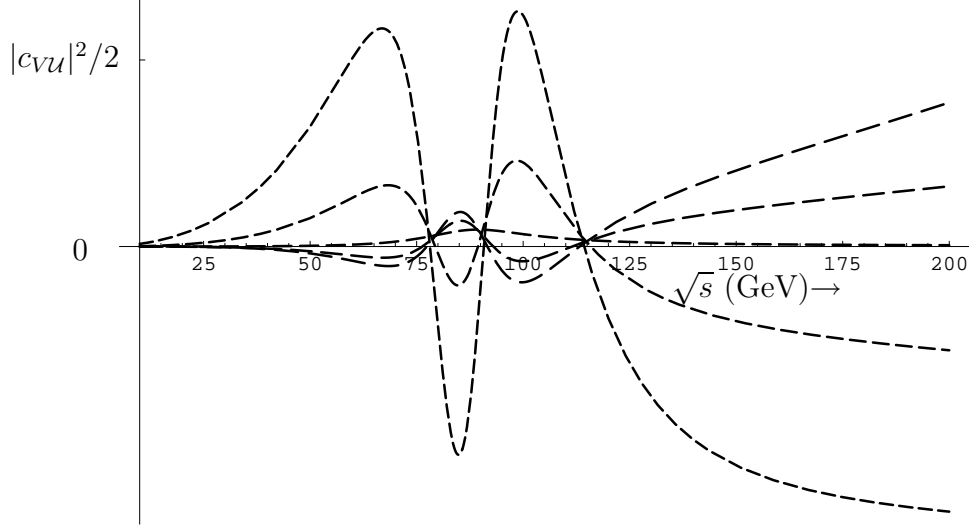


Figure 6: The change in the front-back asymmetry for $e^+e^- \rightarrow \mu^+\mu^-$ versus \sqrt{s} for $d_U = 1.1, 1.3, 1.5, 1.7$ and 1.9 for non-zero c_{VU} and $c_{AU} = 0$. The dash-length increases with d_U .

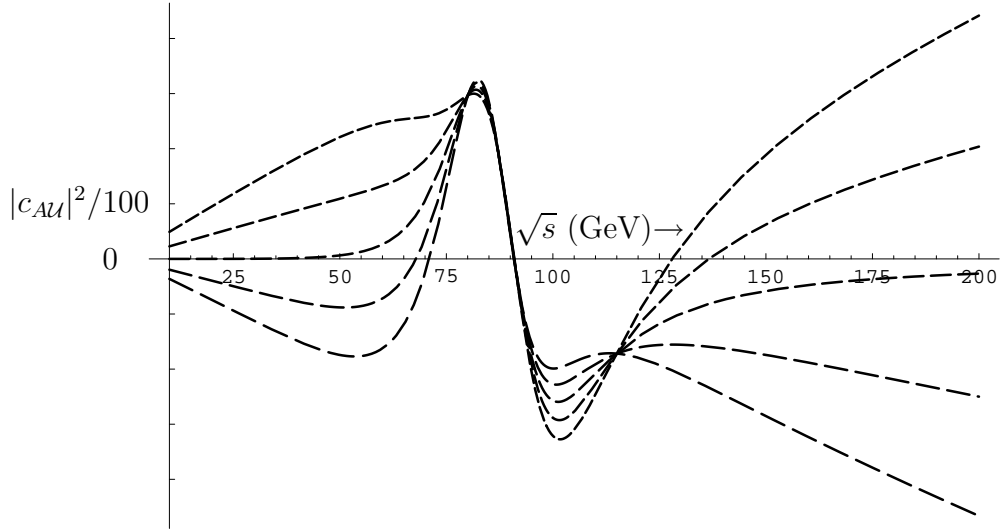


Figure 7: The change in the front-back asymmetry for $e^+e^- \rightarrow \mu^+\mu^-$ versus \sqrt{s} for $d_U = 1.48, 1.49, 1.5, 1.51$ and 1.52 for non-zero c_{AU} and $c_{VU} = 0$. The dash-length increases with d_U .

we still lack an intuitive or even detailed picture of what an unparticle looks like.

Let me close with a couple of more speculative comments. One might argue that the term “propagator” is not particularly felicitous for the unparticle time-ordered product, (3), because the unparticle does not really propagate in the usual way. It is also worth noting the connection between this analysis and the more confusing issue of unparticle decay. There is a sense in which the unparticle exchange amplitude that we have used in our analysis is associated with unparticle production and decay. But in the process we have studied in this note, the decay process is masked by the leading order (and therefore larger) interference

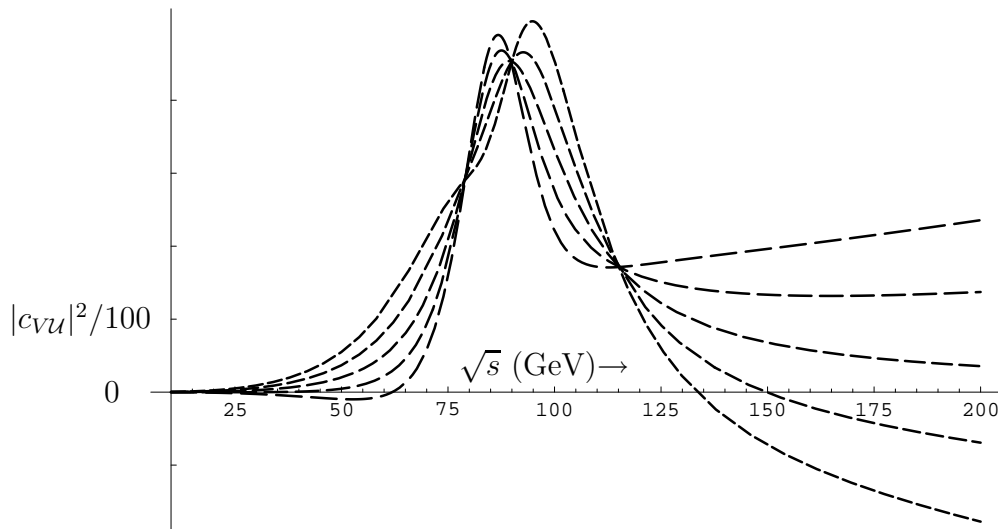


Figure 8: The change in the front-back asymmetry for $e^+e^- \rightarrow \mu^+\mu^-$ versus \sqrt{s} for $d_U = 1.48, 1.49, 1.5, 1.51$ and 1.52 for non-zero c_{VU} and $c_{AU} = 0$. The dash-length increases with d_U .

term. And as with the term “propagator,” the term “decay” may be a little misleading for an unparticle because it suggests that the particle was propagating over an large distance before it decayed. I hope to return to these deliciously confusing issues in a future publication.

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References

- [1] H. Georgi, “Unparticle physics,” [hep-ph/0703260](#).
- [2] E. Eichten, K. D. Lane, and M. E. Peskin, “New tests for quark and lepton substructure,” *Phys. Rev. Lett.* **50** (1983) 811–814.
- [3] T. Banks and A. Zaks, “On the phase structure of vector - like gauge theories with massless fermions,” *Nucl. Phys.* **B196** (1982) 189.
- [4] K. Cheung, W.-Y. Keung, and T.-C. Yuan, “Novel signals in unparticle physics,” [arXiv:0704.2588 \[hep-ph\]](#).